MATHEMATICS APPLICATIONS UNIT 3
Section Two:
Calculator-assumed

SOLUTIONS

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	10	10	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **ten (10)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

(a) The first three terms, in order, of a geometric sequence are 1400, 980 and 686.

    (i) Deduce a rule for the nth term of this sequence. (2 marks)

Solution
$r = 980 \div 1400 = 0.7$
$T_n = 1400(0.7)^{n-1}$
Specific behaviours
✓ calculates ratio
✓ writes rule

    (ii) Calculate the 5th term of the sequence. (1 mark)

Solution
$T_5 = 336.14$
Specific behaviours
✓ exact value of term

(b) The first three terms, in order, of an arithmetic sequence are 2.4, 5.5 and 8.6.

    (i) The rule for the nth term of this sequence is $T_n = an + b$. Determine the values of a and b. (3 marks)

Solution
$d = 5.5 - 2.4 = 3.1$
$T_n = 2.4 + (n-1)(3.1)$ $\quad = 3.1n - 0.7$
$a = 3.1, \qquad b = -0.7$
Specific behaviours
✓ calculates difference
✓ writes rule
✓ simplifies and states values

    (ii) Calculate the 177th term of the sequence. (1 mark)

Solution
$T_{177} = 548$
Specific behaviours
✓ exact value of term

See next page

Question 10   **(8 marks)**

A sequence of four connected graphs is shown below.

G_1 G_2 G_3 G_4
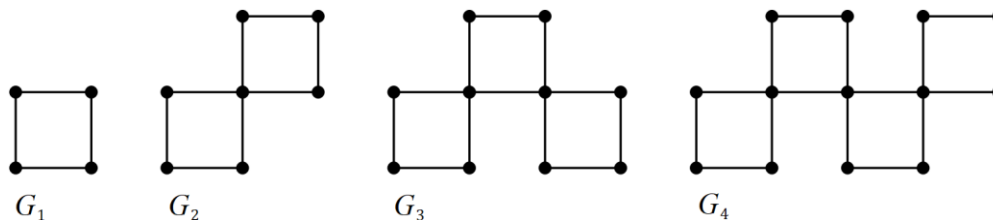
(a) Complete the missing entries in the table below, where the vertex sum is the sum of the degrees of all the vertices in a graph.   (2 marks)

Graph (n)	1	2	3	4
Vertices (V)	4	7	10	13
Vertex sum (S)	8	16	24	32

Solution
See table
Specific behaviours
✓ correct vertices, ✓ correct vertex sums

Assume that the sequence of graphs continues indefinitely.

(b) A graph in the sequence has 10 faces. What is its vertex sum?   (2 marks)

Solution
$10 - 1 = 9 \Rightarrow G_9$
$S_9 = 8 \times 9 = 72$
Specific behaviours
✓ indicates graph number
✓ vertex sum

(c) Deduce the nth term rule for V_n, the number of vertices in graph n.   (2 marks)

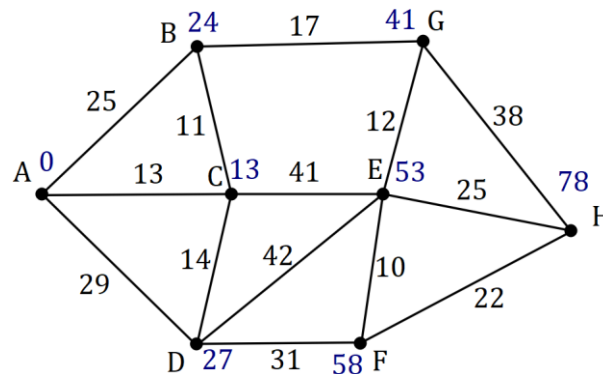| Solution |
|---|
| $V_n = 4 + (n-1)(3)$ |
| $= 3n + 1$ |
| **Specific behaviours** |
| ✓ indicates first term and difference |
| ✓ correct rule |

(d) A graph in the sequence has 343 vertices. Determine the vertex sum of this graph.   (2 marks)

Solution
$3n + 1 = 343 \Rightarrow n = 114$
$S_{114} = 114 \times 8 = 912$
Specific behaviours
✓ uses rule from (c) to find n
✓ vertex sum

Question 11 **(8 marks)**

The graph below represents a network of cycle tracks. The weight on each edge is the length, in km, of that track.

(a) State the length of the shortest route from G to D. (2 marks)

Solution
$GBCD = 17 + 11 + 14 = 42$ km
Specific behaviours
✓ indicates correct route
✓ length of shortest route

(b) Determine the shortest route from A to H, stating the route and its length. (4 marks)

Solution
$ACBGEH = 13 + 11 + 17 + 12 + 25 = 78$ km
Specific behaviours
✓ any route from A to H
✓ correct length of indicated route
✓ shortest route
✓ length of shortest route

(c) The section of track between A and C is closed for repairs and cannot be used. What effect, if any, does this closure have on the length of the shortest route from A to H?

(2 marks)

Solution
Increase the length by 1 km to 79 km ($ABGEH$)
Specific behaviours
✓ indicates an increase
✓ correct length (or increase)

Question 12 **(10 marks)**

A public relations company was tasked with determining whether a person's support for a sugary drinks tax could be associated with their interest in the news.

The company carried out a telephone survey, where people could respond to two questions as shown in the following table:

Question	Choice of response
Are you interested in the news?	Yes or No
Do you support a sugary drinks tax?	Yes, No or Undecided

The responses to the telephone survey are summarised in this table:

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes	667	382	210
	No	157	97	102

(a) Calculate the number of people who

   (i) answered no to being interested in the news. (1 mark)

Solution
$157 + 97 + 102 = 356$
Specific behaviours
✓ number

   (ii) responded to the survey. (1 mark)

Solution
$1259 + 356 = 1\,615$
Specific behaviours
✓ number

(b) If there was no association between interest in the news and support for a sugary drinks tax, should the company expect a smaller percentage of those who support a sugary drinks tax to be interested in the news compared to those who do not support a sugary drinks tax? Explain your answer. (2 marks)

Solution
No. If there was no association then the percentages would be the same.
Specific behaviours
✓ indicates no
✓ explanation indicating same percentages

(c) Complete the two-way table below to show the associated **row** percentages for the previous table, rounding percentages to the nearest whole number. (3 marks)

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes	53%	30%	17%
	No	44%	27%	29%

Solution
See table
Specific behaviours
✓ first row, ✓ second row, ✓ rounds correctly

(d) What percentage of those who are not interested in the news support a sugary drinks tax? (1 mark)

Solution
44%
Specific behaviours
✓ percentage from table

(e) In the context of the task they were given, how should the public relations company interpret the responses to their survey? (2 marks)
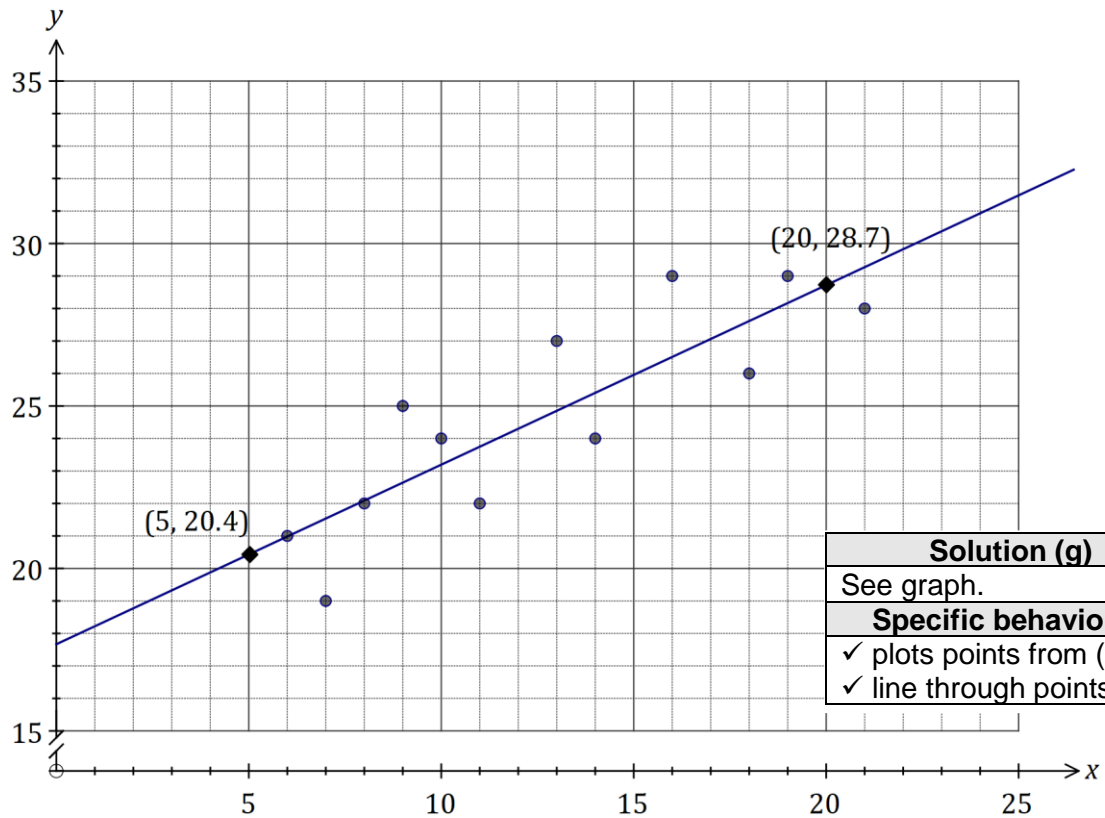
Solution
The PR company should conclude that an association exists between interest in the news and support for a sugary drinks tax.
For example, 53% of those interested in the news supported a sugary drinks tax compared to only 44% of those who were not interested in the news.
Specific behaviours
✓ identifies an association exists
✓ justifies with reference to different percentages in at least one column

Question 13 **(15 marks)**

A tomato grower added varying amounts of a liquid fertiliser (x ml) to the irrigation systems of twelve greenhouses and observed the resulting yield of tomatoes per plant (y kg). A sample of the data recorded is shown in the table and scatterplot below.

x (mL)	6	7	8	9	10	11	13	14	16	18	19	21
y (kg)	21	19	22	25	24	22	27	24	29	26	29	28

Solution (g)
See graph.
Specific behaviours
✓ plots points from (f)
✓ line through points

(a)   Name the explanatory variable. (1 mark)

Solution
x (ml of liquid fertiliser)
Specific behaviours
✓ correct variable

(b)   For this data, calculate

(i)   the correlation coefficient. (1 mark)

Solution
$r = 0.845$
Specific behaviours
✓ value that rounds to 0.85

(ii)   the values of a and b in the equation of the least-squares line $y = ax + b$.

(2 marks)

Solution
$a = 0.553, \qquad b = 17.67$
Specific behaviours
✓ gradient (a)
✓ y-intercept (b)

See next page

(c)   What percentage of the variation in the yield per plant can be explained by the variation in the amount of liquid fertiliser added? (1 mark)

Solution
$r^2 = 0.7136$, which is 71%.
Specific behaviours
✓ states correct percentage

(d)   If the amount of liquid fertiliser added to the irrigation system in a greenhouse was increased by one millilitre, what increase in the yield of tomatoes per plant can be expected? Explain your answer. (2 marks)

Solution
An increase of 0.553 kg per plant. This value if the gradient of the least-squares line.
Specific behaviours
✓ value
✓ indicates use of gradient

(e)   If no liquid fertiliser was used, what yield of tomatoes per plant does the linear model predict? (1 mark)

Solution
A yield of 17.7 kg.
Specific behaviours
✓ uses value of y-intercept

(f)   Use the equation of the least-squares line to calculate the value of y when $x = 5$ and when $x = 20$. (2 marks)

Solution
$x = 5,$ $y = 20.4$
$x = 20,$ $y = 28.7$
Specific behaviours
✓ one correct value
✓ both correct values

(g)   Use your answers to part (f) to draw the least-squares line on the scatterplot. (2 marks)

(h)   Estimate the yield of tomatoes per plant when 24 ml of liquid fertiliser is added to the irrigation system and comment on the reliability of this value. (3 marks)

Solution
$x = 24$ ml, $y = 30.9$ kg per plant
Estimate is unreliable as despite strong correlation it involves extrapolation.
Specific behaviours
✓ any value that rounds to 31 kg
✓ considers correlation
✓ considers extrapolation

Question 14   **(10 marks)**

The roads in a suburb are represented in the graph below, where the number on each edge is the length, in km, of the road.
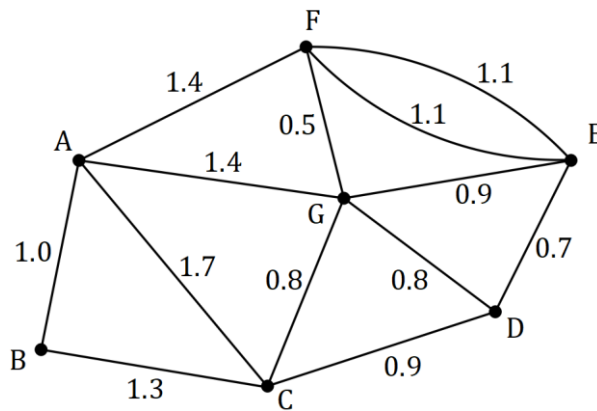
(a)   The network contains a semi-Eulerian trail that ends at G.

        (i)   Explain what semi-Eulerian means.   (2 marks)

Solution
Trail along all edges just once, ending at different vertex to start.
Specific behaviours
✓ all edges once, ✓ different start and finish vertices

        (ii)   At which vertex must the trail start?   (1 mark)

Solution
Vertex D
Specific behaviours
✓ correct vertex

        (iii)   Determine the number of edges in the trail.   (1 mark)

Solution
13 edges
Specific behaviours
✓ correct number

        (iv)   How many times does the trail pass through vertex C?   (1 mark)

Solution
2 times
Specific behaviours
✓ correct number

(b) A worker needs to leave G, travel along each road once to inspect its surface and then return to G. Determine the minimum distance the worker must travel.   (3 marks)

Solution
Sum of weights: 13.6 km
Will end trail at F and then return 0.8 km to G
Minimum distance: $13.6 + 0.8 = 14.4$ km

| Specific behaviours |
| --- |
| ✓ indicates sum of all weights |
| ✓ indicates distance to return to G |
| ✓ correct minimum distance |

(c) Another worker needs to walk along each road twice, once on each side of the road, as they deliver advertising pamphlets to houses. Determine, with justification, the minimum distance this worker must travel if they start and finish at A and the width of the roads is ignored.   (2 marks)

Solution
Each edge is duplicated, so all vertices even and graph is now Eulerian - can start at any vertex and return after visiting all edges.
Minimum distance is double sum of weights: $2 \times 13.6 = 27.2$ km

Specific behaviours
✓ reasoning
✓ correct distance

Question 15   **(8 marks)**

A linear model was fitted to a set of data, resulting in a correlation coefficient of $r = 0.94$ and a least-squares line with equation $\hat{y} = 5.16 + 0.16x$. A residual plot for the linear model is shown below.
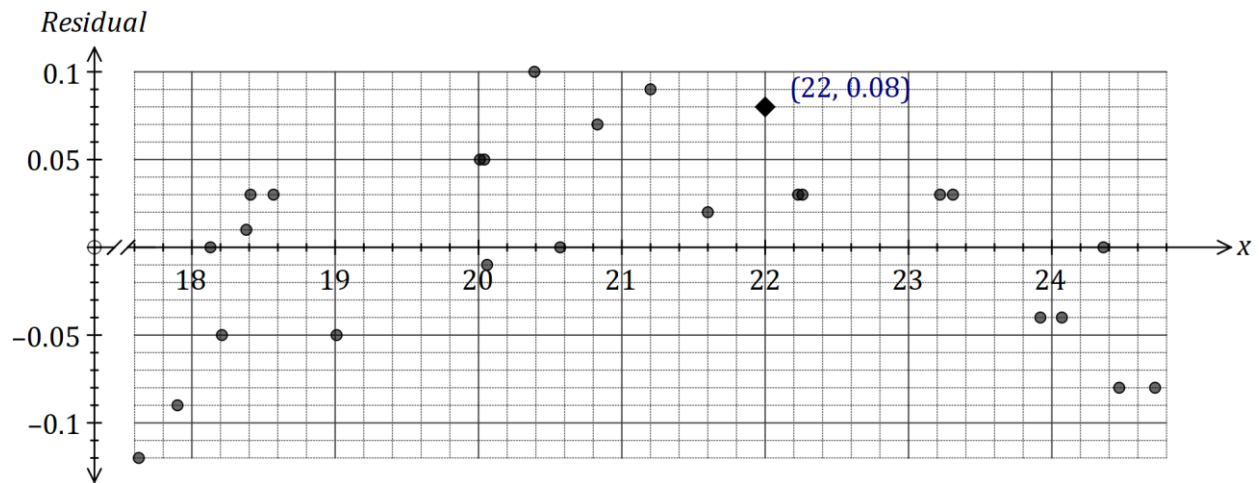
(a) Calculate, and add to the plot above, the residual for the point $x = 22$, $y = 8.76$.

(3 marks)

Solution
$\hat{y} = 5.16 + 0.16(22) = 8.68$
Residual $= 8.76 - 8.68 = 0.08$
Specific behaviours
✓ calculates $\hat{y}$
✓ calculates residual
✓ plots residual

(b) Use the residual plot to comment on the appropriateness of fitting a linear model to the data.

(2 marks)

Solution
A pattern is evident in the residual plot (as x values increase, residuals start mostly negative, then positive and then negative again) and so a linear model is not appropriate.
Specific behaviours
✓ not appropriate
✓ reason

(c) Determine the y-coordinate of the point with a residual of 0.07 on the above plot.

(3 marks)

Solution
$x = 20.8$
$\hat{y} = 5.16 + 0.16(20.8) = 8.488$
$y - 8.488 = 0.07 \Rightarrow y = 8.558$
Specific behaviours
✓ indicates x-coordinate
✓ calculates $\hat{y}$
✓ calculates y

Question 16 **(9 marks)**

The temperature, T °C, of an industrial oven n minutes after it is turned on can be modelled by

$$T_{n+1} = 0.92T_n + 19.2, \qquad T_0 = 15$$

(a) Use the recurrence relation to complete the table of values below, rounding the temperature to the nearest °C. (2 marks)

n	0	10	20	30	40
T_n	15	142	198	222	232

Solution
See table
Specific behaviours
✓ at least 3 values correct, ✓ all values correct

(b) Sketch a graph of the temperature of the oven for the first 75 minutes on the axes below. Make sure you add a suitable scale to the vertical axis. (4 marks)
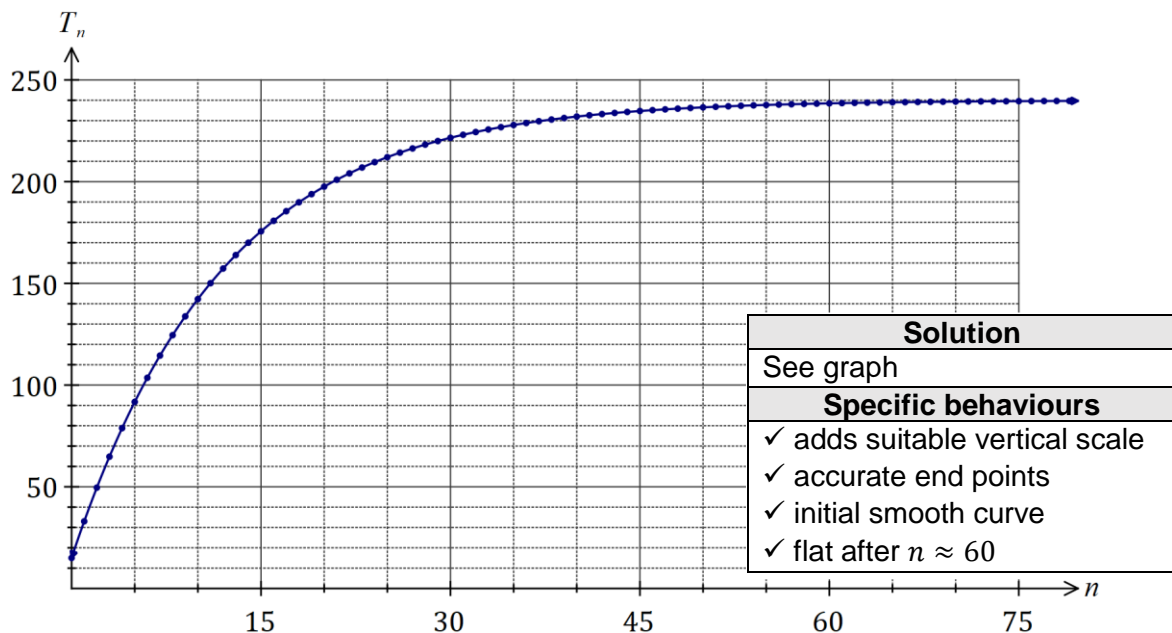
Solution
See graph
Specific behaviours
✓ adds suitable vertical scale
✓ accurate end points
✓ initial smooth curve
✓ flat after $n \approx 60$

(c) The manufacturer claims that the oven will reach within 10 °C of its maximum temperature within 30 minutes of being turned on. Comment on this claim. (3 marks)

Solution
Maximum temperature is 240°C.
Temperature exceeds 230°C after 38 minutes.
Claim is false.
Specific behaviours
✓ indicates maximum temperature
✓ states time reaches within 10°C
✓ comment on claim

Question 17   **(13 marks)**

When Atarcoin, a new cryptocurrency was launched, one Atarcoin was valued at $3.00. After one week of trading, the value of Atarcoin had increased to $3.60, and after another week had increased to $4.32.

(a)   Show that the value of Atarcoin increased by 20% each week.   (2 marks)

Solution
$3.6 \div 3 = 1.2$, $4.32 \div 3.6 = 1.2 \Rightarrow 20\%$ increase
or
$3 \times 1.2 = 3.60$, $3.60 \times 1.2 = 4.32$
Specific behaviours
✓ shows one ratio (or multiple)
✓ shows both ratios (or multiples)

The value of Atarcoin, V_n in dollars, n weeks after its launch date, can be modelled by the recurrence relation $V_{n+1} = 1.2V_n$, $V_0 = 3$.
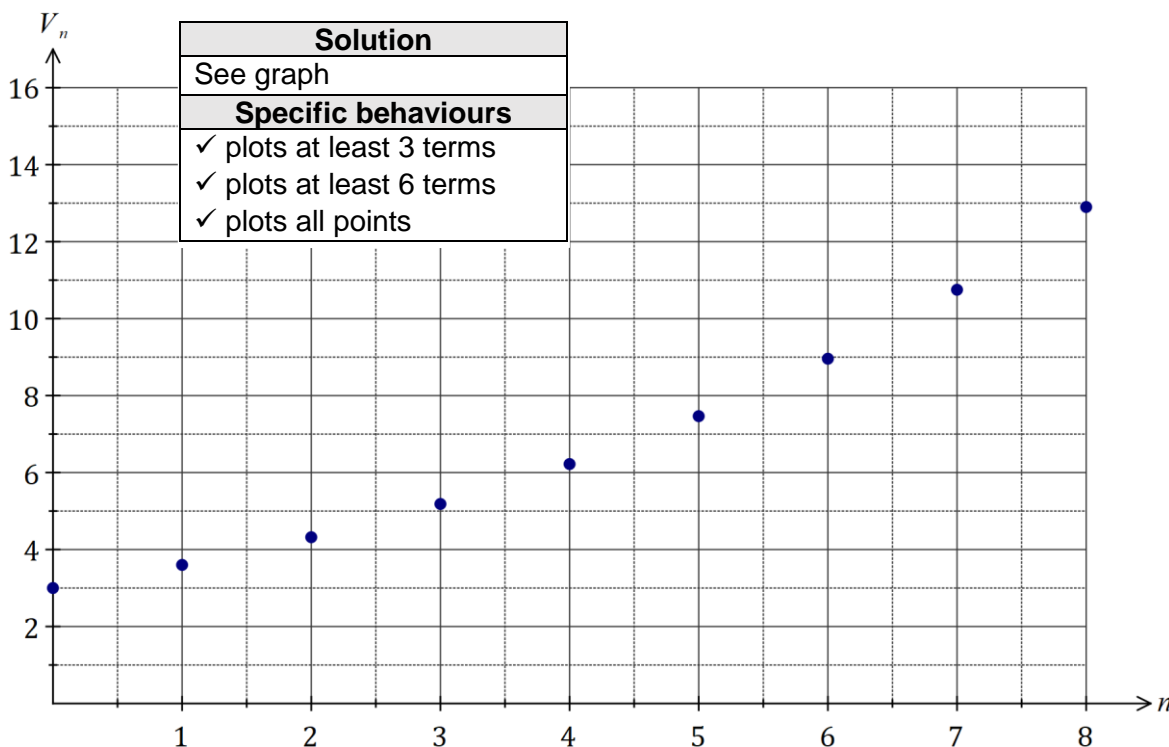
(b)   Calculate the value of Atarcoin 11 weeks after its launch date.   (1 mark)

Solution
$V_{11} = \$22.29$
Specific behaviours
✓ value

(c)   At the end of which week did the value of Atarcoin first exceed $150?   (1 mark)

Solution
End of week 22.
Specific behaviours
✓ week

(d)   Graph V_n against n on the axes below.   (3 marks)

Solution
See graph
Specific behaviours
✓ plots at least 3 terms
✓ plots at least 6 terms
✓ plots all points

The value of Atarcoin peaked at the end of week 35, and from that time onwards, its value fell by 40% each week.

(e) Determine the value of Atarcoin at the end of week 36. (2 marks)

Solution
$V_{35} = 1772.00$
$T_{36} = V_{35} \times 0.6 = \$1\,063.20$
Specific behaviours
✓ peak value
✓ value one week later

(f) Ignoring any fees involved in buying and selling a cryptocurrency, determine the profit or loss made by a person who bought 150 Atarcoins four weeks after their launch, held them for 35 weeks and then sold them all. (4 marks)
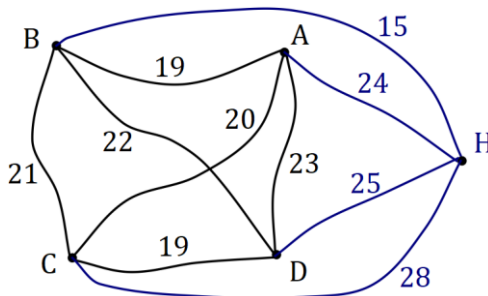
Solution
$V_4 = 6.22$
$35 - 4 = 31, 35 - 31 = 4$
Sells 4 weeks after peak
$T_{n+1} = 0.6T_n, \qquad T_0 = 1772$
$T_4 = 229.65$
Profit $= 150 \times (229.65 - 6.22)$
$= 150 \times 223.43$
$= \$33\,514.50$
Specific behaviours
✓ value when bought
✓ week sold
✓ value when sold
✓ profit

Question 18   **(10 marks)**

Oliver is visiting a city that has four museums: A, B, C and D. The weights on the edges of the following graph represent the time, in minutes, that it takes to walk between the museums.

Solution (c)
See graph
Specific behaviours
✓ correct edges
✓ correct weights

(a) List, in the order visited, a set of vertices that form a Hamiltonian cycle in the graph.

                                          (2 marks)

Solution
$ABCDA$
Specific behaviours
✓ all vertices listed just once
✓ start and finish at same vertex

(b) Determine the shortest time it would take to leave C, walk to the other museums and return to C.   (2 marks)

Solution
$CDBAC = 19 + 22 + 19 + 20 = 80$ m
Specific behaviours
✓ indicates vertices on a cycle from C
✓ shortest time

The time to walk from Oliver's hotel, H, to museums A, B, C and D is $24, 15, 28$ and 25 minutes respectively.

(c) Add vertex H and this information to the graph above.   (2 marks)

(d) Oliver plans to leave his hotel at 9.30 am, visit all the museums and then return to his hotel at 3.30 pm the same day. Determine the maximum total time he can spend inside the museums if he walks between them and describe the route he should take.

                                          (4 marks)

Solution
$HBACDH = 15 + 19 + 20 + 19 + 25 = 98$ m
$6 \text{ h} - 1 \text{ h } 38 \text{ m} = 4 \text{ h } 22 \text{ m} \quad (262 \text{ m})$
Specific behaviours
✓ lists possible route
✓ lists quickest route
✓ indicates quickest time
✓ calculates time in museums using h and m

Supplementary page

Question number: _____

Supplementary page

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Supplementary page

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